Final

Evolution of Nonlinear Alfvén Waves in the Interplanetary Medium

B. Buti [1], V.L. Galinski [2], V.I. Shevchenko [3], G.S. Lakhina [1], B.T. Tsurutani [1], P. Diamond [4] and M.V. Medvedev [4]

1. Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109.
2. Scripps Institution of oceanography, University of California, San Diego, LaJolla, CA 92093.
3. Department of Electrical and Computer Engineering, University of California, San Diego, LaJolla, CA 92093.
4. Department of Physics, University of California, San Diego, LaJolla, CA 92093.

Abstract

Nonlinear evolution equation for Alfvén waves, propagating in streaming plasmas with nonuniform densities and inhomogeneous magnetic fields, is obtained by using reductive perturbation technique. Numerical solution of this equation, which is a modified derivative nonlinear Schrödinger (MDNLS) equation, shows that the inhomogeneities exhibit their presence as the dissipation. The evolution, of long wave length Alfvénic fluctuations, shows that the wave gets steepened and accelerated as it propagates away from the sun in interplanetary medium. High frequency radiations are also observed in our simulations. The spectral index, in the frequency range 3×10^{-4} - 10^{-2}, is found to vary from 1.6 to 2.6 with the heliospheric distance changing from 0.5 AU to 0.9 AU. This is in very good agreement with the observed spectra for the solar wind turbulence.

Introduction

Large amplitude Alfvén waves have been observed in solar wind, planetary bow shocks, interplanetary shocks and environment of comets (Belcher and Davis, 1971; Scarf et al., 19S6), Mariner 5 data (Belcher and Davis, 1971) showed that these waves were propagating out ward from the sun wit h the wave-lengt h range from 10^3 to 5×10^6 kms. The power spectra for the magnetic field, in the frequency range 1.5×10^{-4} - 4×10^{-2} Hz, had the frequency dependence ranging from $f^{-1.5}$ to $f^{-2.2}$. Bavassano et al. (1982) analyzed the magnetic field data of HEL1OS 1 and 2 for the heliocentric distances between 0.3 and 0.9 AU. They also found that in the frequency range '2. S $\times 10^{-4}$ - 8.3×10^{-2} Hz the spectral index (α), of the power spectra, increases with the heliocentric distance. Average values of α range between 1.2 - '2.0 for 0.3

0.9 AU. The radial dependence of α depends on the frequency. α is found to be smaller for higher frequencies. Recently Marsch and Tu (1996) studied the evolution of α using Ulysses data. They found that the evolution of α is slower for the polar flows. Moreover the data showed break-point in the frequency spectrum. According to the data from HELIOS 1 and 2 and Ulysses, break-poirit moves to lower frequency with an increase in the heliocentric distance. From Ulysses data, Tsurutani et al. (1996) showed that the spectral index hardly changes in the frequency range 10-'- 10^{-1} at high heliographic latitudes.

Nonlinear evolution of Alfvén waves in uniform plasmas is governed by Vector Derivative Nonlinear Schrödinger (VDNLS) arid DNLS equations (Kennel et al., 19SS ; Buti, 1990). Hada et al, (1990) had included the dissipative effects whereas the kinetic effects were incorporated into the evolution equation by Register (1971), Mjolhus and Wyller (1986, 1988), Spangler (1989, 1990) and Medvedev and Diamond (1996). Medvedev et al. (1 997) showed that the inclusion of heat flux modelled as ion-Landau damping in the Kinetic Nonlinear Schrödinger (KNLS) equation leads to evolution of Alfvén waves into the rotational discontinuities. All these investigations, however, are restricted to homogeneous systems but most of the plasmas, where nonlinear Alfvén waves have been observed, have inhomogeneous densities as well as magnetic fields. Buti (1991) had rederived the governing equation for these waves in inhomogeneous plasmas and had shown that the inhomogeneities lead to acceleration (deceleration) of the solitary Alfvén waves depending upon the direction of propagation vis-a-vis the density gradients (Lakhina et al., 1990). In this analysis, even though no explicit assumption about homogeneity of the magnetic field was made but because of the slab geometry used, implicitly the field considered was homogeneous. In the present paper, we have removed this implicit restriction by incorporating the spherical geometry. The evolution equation derived holds good for any arbitrary inhomogeneities. Numerical solution of this equation exhibits most of the observed features (the wave steepening, emission of high frquency radiations etc.) of the solar wind. Moreover the predicted power spectra have spectral indices which are in good quantitative agreement with the spectral indices of the observed solar wind turbulence.

Evolution of Nonlinear Alfvén Waves

The nonlinear equations governing Alfvén waves propagating in the radial direction are two-fluid actuations and the generalized Ohm's law (Kennel et al., 19SS; Buti, 1990, 1991). On using these equations in spherical co-ordinates and assuming no variations along θ and ϕ directions, we have derived the evolution equation (details of the derivation will be reported in the detailed paper later). For the derivation, we have used the reductive perturbation method and have followed the procedure

Buti (1991). For the spherical solar wind, we obtain the following modified DNLS equation:

$$\frac{\partial B}{\partial \eta} + \frac{3\,U}{2\,V\,\eta}\,B + \frac{B}{4\,V\,(V-U)}\,\frac{\partial}{\partial \eta}\left(V^2 - U^2\right) \tag{1}$$
$$+\frac{(V-U)}{4\,B_0^2(\eta)\,V^2\,(1-\beta(\eta))}\,\frac{\partial}{\partial \xi}\left(B\mid B\mid^2\right) + \frac{i\,(V-U)^2}{2\,V^3\,B_0(\eta)}\,\frac{\partial^2\,B}{\partial \xi^2} = 0,$$

where B = (f?$_e$ + $i\,B_\phi$) , $\eta = \epsilon^2$ r, $\xi = \epsilon\,[\,\int dr/V(r) - t],\epsilon$ is the stretching parameter, r *is* the radial distance, U is the streaming plasma velocity; V is the phase velocity of Alfvén wave, $\beta(\eta)$ is the plasma β, i.e., the ratio of kinetic pressure to magnetic pressure and $B_0(\eta)$ is the ambient magnetic field. In Eq. (1), B is normalized to $B_0(r = r_o)$, U and V are normalized to $V_{A0} = B_0(r = r_0)/[4\pi\rho_0(r = r_0)]^{1/2}$, the Alfvén velocity, t to $\Omega_{io} = \Omega(r = r_o)$, the ion cyclotron frequency and r to V_{A0}/Ω_{io}. The equilibrium density $\rho_0(r)$ and the magnetic field *Be(r)* must satisfy the conditions:

$$B_0(r)\ 7'^2 = const \tag{2}$$

and

$$\rho_0(r)\quad U(r)\,r^2 = const \tag{3}$$

In deriving Eq. (1), we have taken the wave propagation as well as the ambient magnetic field along the radial direction. This equation is valid for arbitrary inhomogeneities. We may note that for nonstreaming uniform plasmas i.e., for U = O and $\rho_0(r) = 1, V \to 1$ and Eq. (1) reduces to the well known DNLS equation. It is interesting to observe that this modified DNLS (Eq (l)), besides having additional two linear terms in B, has variable co-efficients for the nonlinear and the dispersive terms. Because of these complicated variable co-efficients, it is not possible to find an analytical solution to Eq. (1). We have solved this equation numerically by using Pseudo-Spectral method (Galinsky et al., 1990). In our numerical scheme, magnetic field is represented as the Fourier-series in ξ and the radial evolution is obtained by using the predictor - corrector scheme with all linear terms expressed in an implicit form and the nonlinearity being extrapolated on the predictor stage.

For the numerical solution, we have considered two cases. 1) Evolution of an initial Alfvén soliton which is the exact solution of the DNLS equation and 2) evolution of an initially circularly polarized wave. For the first case we take

$$B\,(\xi,\ r_0)\ =\ \frac{B_{max}\ e^{i\theta\,(\xi)}}{cosh^{1/2}\ \psi}, \tag{4}$$

with

$$\psi = (\xi - L/2)\, B_{max}^2 / (2\,\delta), \tag{5}$$

$$\theta(\xi) = \frac{3}{2} \tan^{-1}(\sinh \psi) \tag{6}$$

and

$$\delta = \frac{2}{V}(1 - \beta)(V - U). \tag{i'}$$

B_{max} in Eq.(4) is the amplitude of the initial soliton normalized to $B_0(r_0)$. We would like to point out that the solution given in Eq.(4) is different than the one we had used earlier (Verheest and Buti,1992; Hada et al., 1990; Mjolhus, 1978). In the soliton solution (Mjolhus, 1978), there are two arbitrary constants κ_0 and ν_0. We had taken V. = O and now for Eq.(4) κ_0 has been taken to be zero. For our reference point, we take $r_0 = R_s$(R_s being the solar radius). The other simulation parameters are : $\beta(r_0) = 0.01$. and $r_0\,\Omega_{i0} / V_{A0} = 10^6$. Fig.1 shows the variation of $B_{max} / B_0(r)$ with the heliocentric distance for different initial conditions. In all the three cases, we see that the magnetic field fluctuations increase with increasing r. This is in agreement with the observations (Klinglesmith,1997). Fig.2a shows the time evolution of the field starting with the soliton amplitude of 0.036 $B_0(R_0 = 0.1\ AU)$, $U_0 = 3.5\ V_{AO}$ and $\beta(R_0) = 0{,}05$. We find that the amplitude goes down as the wave propagates away from the sun. We also see the steepening of the wave and the high-frequency radiations on the leading edges. Steepening is found to increase with the increasing heliospheric distances. The corresponding spectra for the field fluctuations are shown in Fig.2b. It is interesting to note the break in the power spectra. Once again in good agreement with the observations (Bavassano et al.,1982; Marsch and Tu, 1996), we find that the spectral index (α) increase from 1.6 to 2.6 with an increase in the radial distance from 0.5-- 0.9 AU. The break point moves from high to low frequencies as r increases. These calculations are repeated for the solar wind closer to the sun. The results for the magnetic field fluctuations are shown in Fig.3a and the evolution of spectra with heliocentric distance is shown in Fig.3b. Qualitatively the evolution closer to the sun is found to be similar to the one between 0.1- 1 AU.

For the second case i.e., for the initially circularly polarized case, we take

$$B(R_0, t) = 0.01\ [\exp \frac{2\,i\,\pi\,t}{T_{max}} + \exp \frac{4\,i\,\pi\,t}{T_{max}}], \tag{s}$$

with $T_{max} = 19200\ \pi\,\Omega_{i0}^{-1}$. From Fig.4, we see that the evolution in this case is much slower. High frequency radiations are seen only for $r > 0.9$ AU, The spectra (not shown) also do not evolve into power spectra during the periods considered.

Conclusions

Large amplitude Alfvén waves propagating radially outward from the sun are governed by the MDNLS equation. Numerical solution of this equation shows that the Alfvén soliton evolves with space and time and shows features e.g., magnetic field fluctuations, power spectra, wave steepening, which are in very good agreement with the observations of Mariner 5, Helios 1,2 and Ulysses, The spectral indices of the predicted power spectra range between 1.6 -2.6 wheras for the same frequency range the observed spectral indices, from Mariner 5 as well as Helios 1 and 2 data, range between 1.2-2.2.

We would like to emphasize that even though we initially start with a coherent structure like the DNLS soliton, because of inhomogeneities embedded in our MDNLS equation, it evolves into noncoherent/turbulent state. This seems to be a good reason for the non-existence of solitons in the solar wind. The complete integrability property of the DNLS is clearly destroyed by these inhomogeneities. We plan to do the Painléve analysis of the MDNLS equation to determine its stability properties. To cover the entire β range for the solar wind, we plan to include the kinetic effects in our future model.

Acknowledgments. The research conducted at the Jet Propulsion Laboratory, California Institute of Technology, was performed under contract to the National Aeronautics and Space Administration. GSL held a Senior Resident Research Associateship of the National Research Council during this period. The work done at UCSD was supported by NASA Grant NAG W-5157. VLG and VIS would like to thank W. Coles for very useful discussions.

REFERENCES

Bavassano, Il., M. Dobrowolny, F. Mariani and N.F. Ness, Radial evolution of Power Spectra of Interplanetary Alfvénic Turbulence, *Jou. Geophys. Res., 87*, 361 i', 1982.

Belcher, J.W. and L. Davis, Large Amplitude Alfvénic Waves in the Interplanetary Medium, *Jou. Geophys. Res., 76*, 3534, 1971.

Buti, B., *Solar and Planetary Plasma Physics, Ed. B. Buti, World Scientific, Singapore,* 92, *1990.*

Buti, B., Nonlinear Alfvén Waves in Inhomogeneous Plasmas, *Geophys. Res. Lett., 18*, 809, 1991.

Galinsky, V. L., A.V. Khrabrov, and V.I. Shevchenko, The evolution of high amplitude parallel propagating MHD-waves in the vicinity of bow shocks, *Planet. Space Sci., 38*, 1069, 1990.

Hada, T., C. F'. Kennel, B. Buti and E. Mjolhus, Chaos in Driven Alfvén Systems *Phys. Fluids, B2, 2581,* 1990.

Kennel, C. F., B. Buti, T. Hada and R. Pellat, Nonlinear, Dispersive, Elliptically Polarized Alfvén Waves *Phys. Fluids, 31*,1949, *1988.*

Klinglesmith, M., The Polar Solar Wind from 2.5 to 40 Solar Radii: Results of Intensity Scintillation Measurements, *Ph.D. Dissertation, UCSD, 1997.*

Lakhina, G. S., B. Buti and N. I,. Tsintsadze, Radiation from Accelerated Alfven Solitons in Inhomogeneous Plasmas, *Astrophys. Jou., .352, 747, 1990.*

Marsch, E. and C.Y. Tu, Spatial Evolution of the Magnetic Field Spectral Exponent in the Solar Wind: Helios and Ulysses Comparison, *Jou. Geophys. Res., 101, 11149, 1996.*

Medvedev, M.V. and P. II. Diamond, Fluid Models for Kinetic Effects on Coherent Nonlinear Alfvén Waves. I. Fundamental Theory *Phys. Plasmas,* 3, 863, 1996.

Medvedev, M.V. and P.H. Diamond, V.I. Shevchenko and V.L. Galinsky, Dissipative Dynamics of Collisionless Nonlinear Alfvén Wave, *Phys. Rev. Lett.,*, 1997.

Mjolhus, 13., Note on the Modulational Instability of long Alfvén Waves parallel to the Magnetic Field, *Jou. Plasma Phys., 19, 437, 1978.*

Mjolhus, E. and J. Wyller, Alfvén Solitons, *Physica Scripts, 33, 442, 1986.*

Mjolhus, E. and J. Wyller, Nonlinear Alfvén Waves in a Finite - Beta Plasma, *Jou. Plasma Phys.,* 40, 299, 1988.

Register, A., Parallel Propagation of Nonlinear Low-Frequency Waves in High $-\beta$ Plasma, *Phys. Fluids* 14, 2733, 1971.

Scarf, F. L., F.V. Coroniti, C. I'. Kennel, D.A. Gurnett and E.J. Smith, Initial Report of Plasma Wave Observations at Comet Giacobini-Zinner, Science, 232, 382, 1986.

Spangler, S. R., Kinetic Effects of Alfvén Wave Nonlinearity. I. Pondermotive Density Fluctuations, *Phys. Fluids, B1, 1738, 1989.*

Spangler, S. R., Kinetic Effects of Alfvén Wave Nonlinearity. II. 'l'he Modified Nonlinear Wave Equation, *Phys. Fluids, B2, 407, 1990.*

Tsurutani, B. T., C.M. Ho, J.K. Arballo, E.J. Smith, B.E. Goldstein, M. Neugebauer, A. Balogh and W.C. Feldman, Interplanetary Discontinuities and Alfvén Waves at high Heliographic latitudes: Ulysses, *Jou. Geophys. Res., 101, 11027, 1996.*

Verheest, F. and B. Buti, Parallel Solitary Alfvén Waves in Warm Multispecies Beam-Plasma Systems *Jou. Plasma Phys.,* 4?', 15, 1992.

Figure Captions

Fig.1 shows the variation of $B_{max} / B_0(r)$ with the heliocentric distance r / R_s. for 3 cases with $\beta(r_o) = 0.01$. and r_0 C?$_{io}$ / $V_{A0} = 10^6$. For case 1, $B_{max} = 0.026 B_0(r_0)$ and $T_{max} = 64000 \; \pi \Omega_{i0}^{-1}$; case 2, $B_{max} = 0.039 \; B_0(r_o)$ and $T_{max} = 64000 \; \pi \Omega_{i0}^{-1}$ and case 3, $B_{max} = 0.014 B_0(r_o)$ and $T_{max} = 192000 \; \pi \Omega_{i0}^{-1}$,

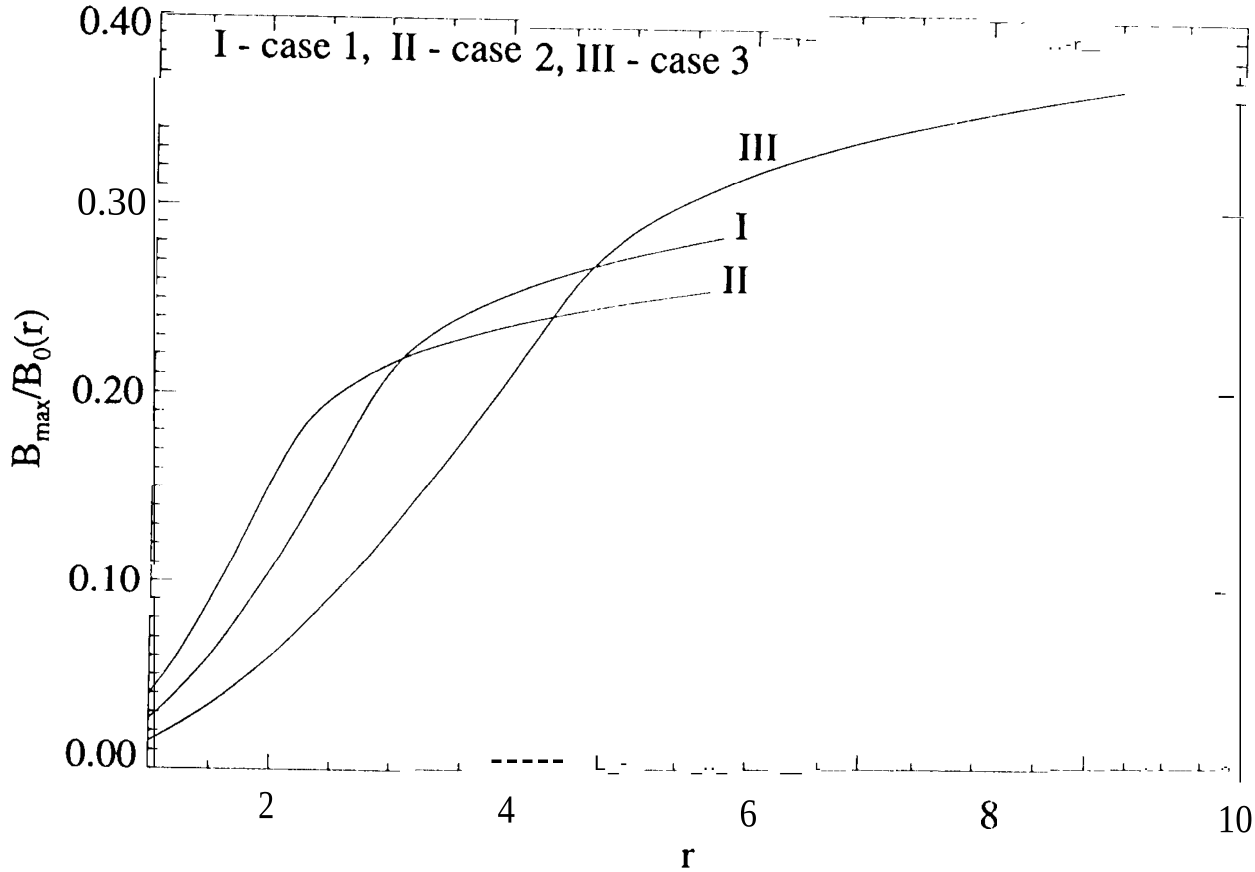
Fig.2a shows evolution of $B / B_0(r_o)$ with t for $B_{max}(R_0) = 0.036$, $R_0 = 0.1$ AU, U_0 – 3.5 V_{AO} and $\beta(r_0) = 0.05$. Curves labelled 1, 2, 3, 4 and 5 correspond to r = 0.1 AU, 0.35 AU, 0.5 AU, 0.7 AU and 0.9 AU.

Fig.2b shows the power spectra for heliospheric distances 0.5 AU, 0.7 AU and 0.9 AU. The parameters used are same a-s Fig.2a.

Fig.3a shows evolution of $B / B_0(R_0)$ with t for $B_{max}(R_0) = 0.014$, $R_0 = R_s, U_0 =$ 0.5 V_{AO} and $\beta(r_0) = 0.01$. Curves labelled 1, 2, 3, 4 and 5 correspond to $r =$ 1 R_s, 3.5 R_s, 5 R_s, 7 R_s and 9 R_s.

Fig.3b shows the power spectra for heliospheric distances 5 R,, 7 R_s and 9 R_s. The parameters used are same as for Fig.3a.

Fig.4 shows the evolution of B / B. (R_0) with t for initially circularly polarized wave for heliospheric distances 0.1 AU, 0.7 AU, 0.9 AU and 1.05 AU. The parameters used are same as for Fig.2a.



I - case 1, II - case 2, III - case 3
III
I
II
0.40
0.30
0.20
0.10
0.00
$B_{max}/B_0(r)$
2
4
6
8
10
r

Fig. 1

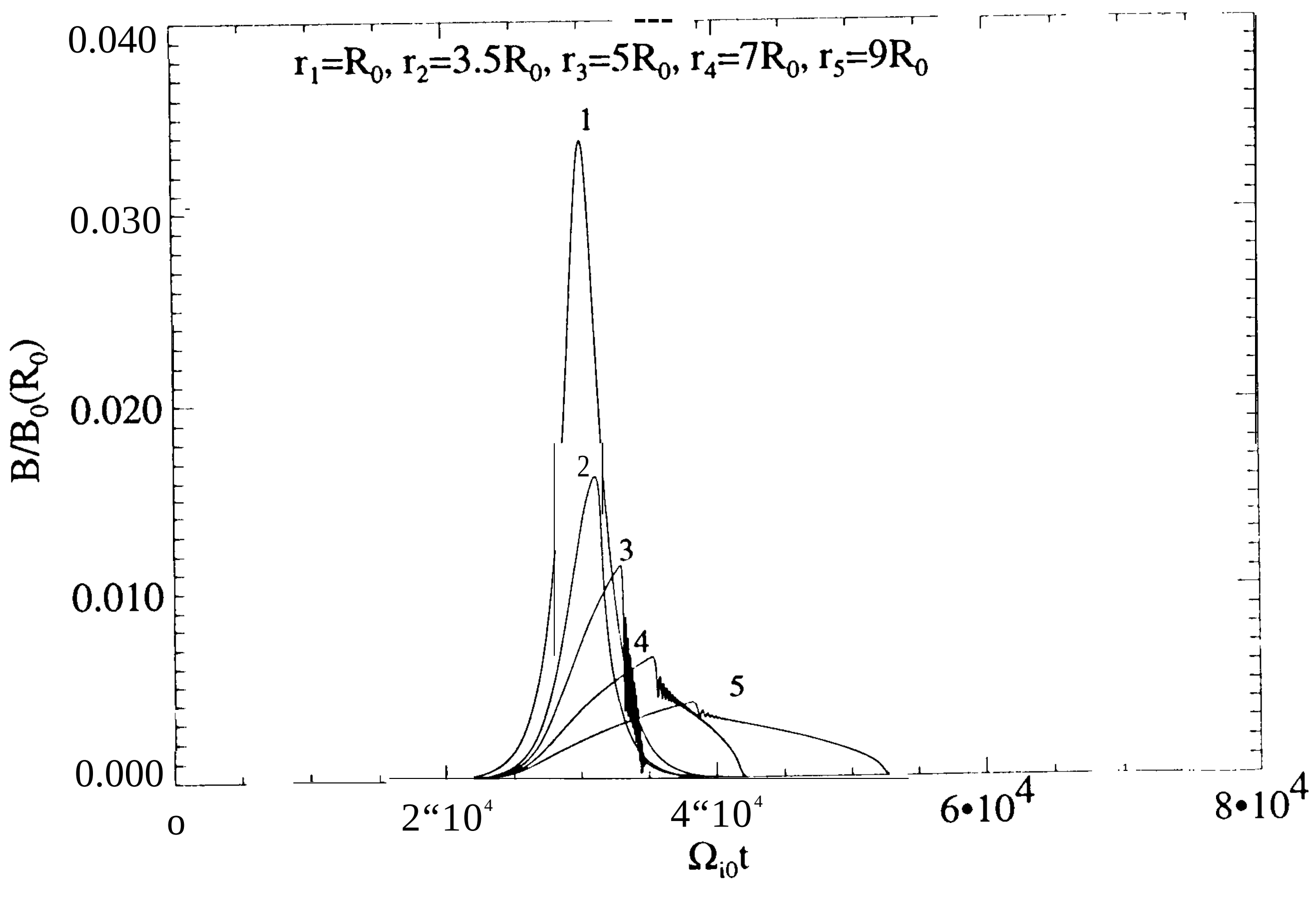
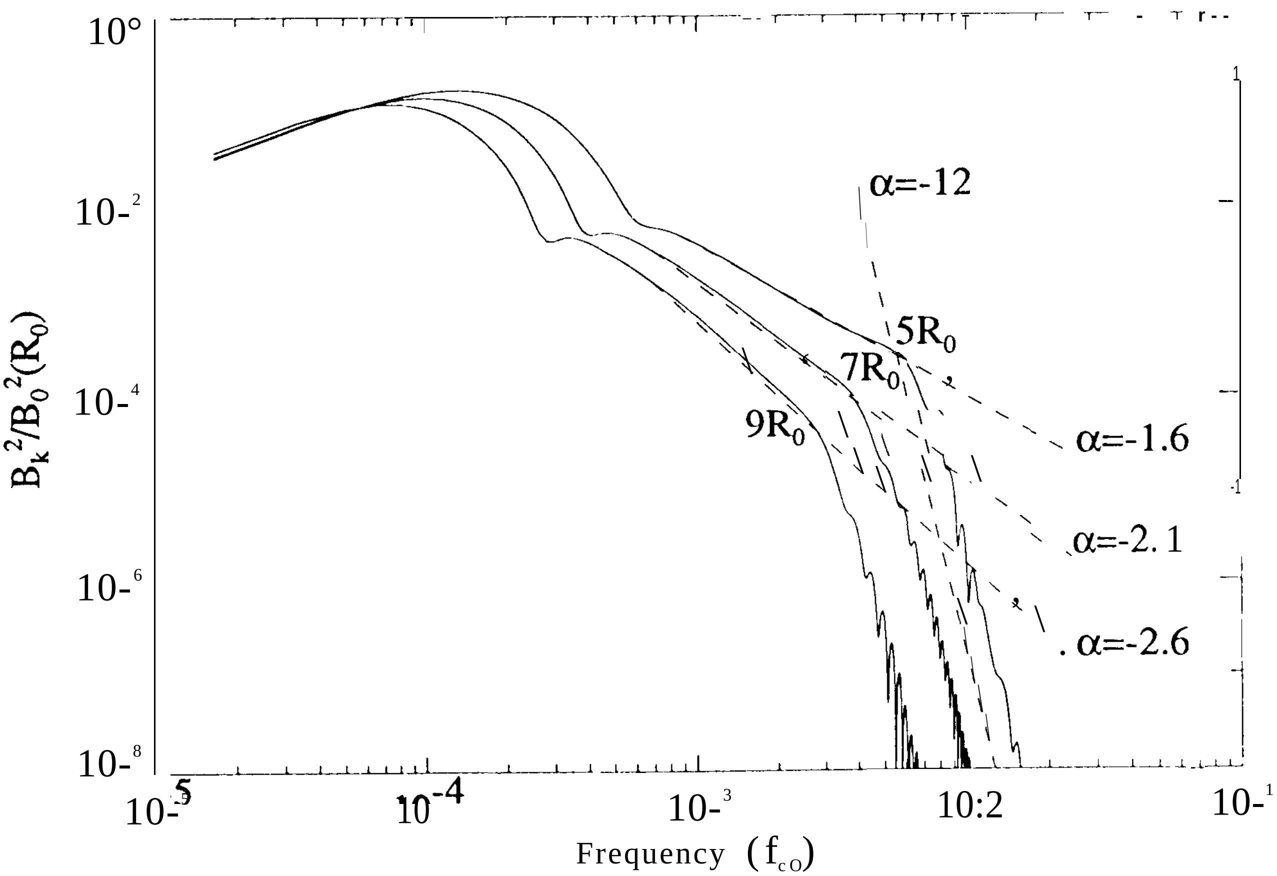
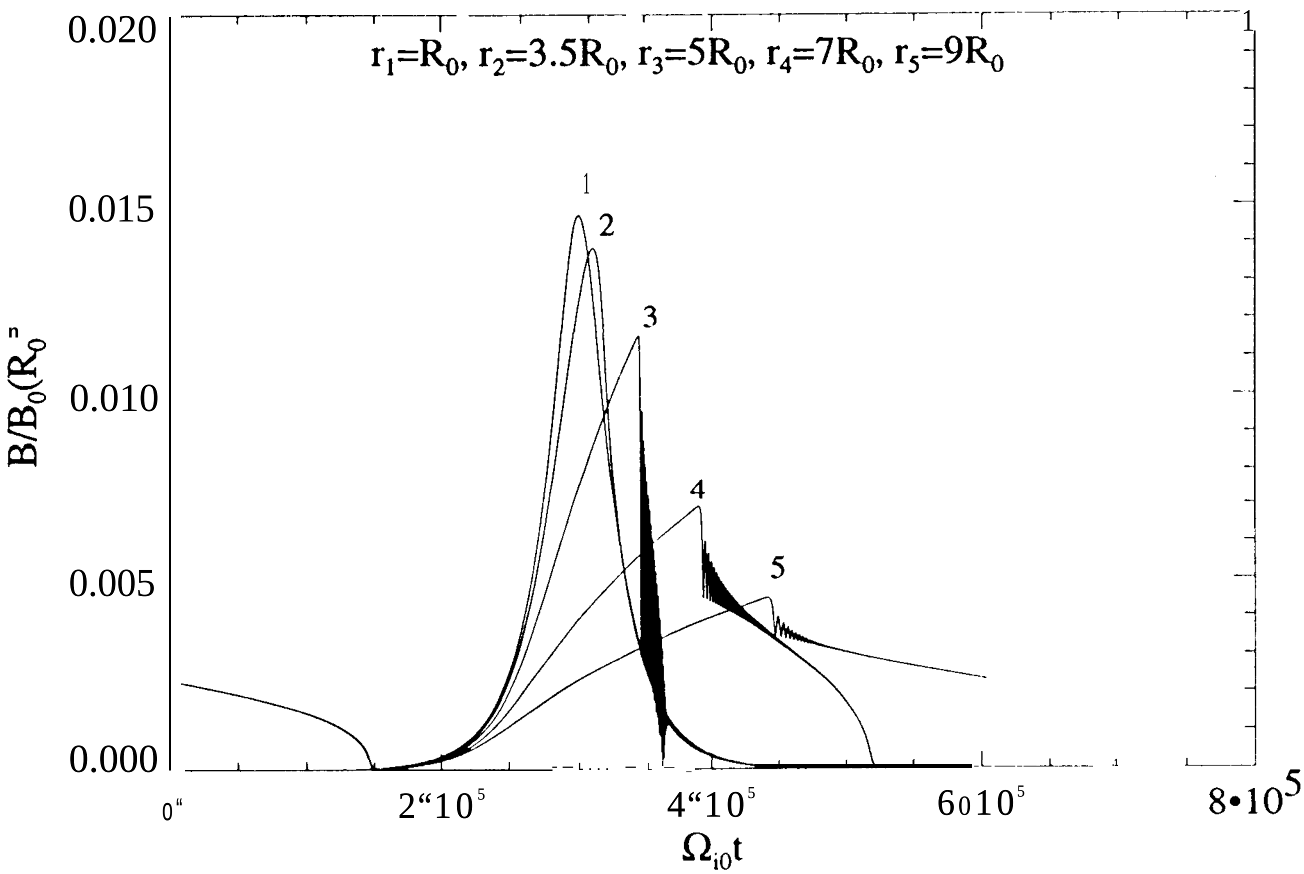

0.040
0.030
0.020
0.010
0.000
B/B0(R0)
r1=R0, r2=3.5R0, r3=5R0, r4=7R0, r5=9R0
1
2
3
4
5
0
2“10⁴
4“10⁴
6•10⁴
8•10⁴
Ωi0t

Fig. 2a



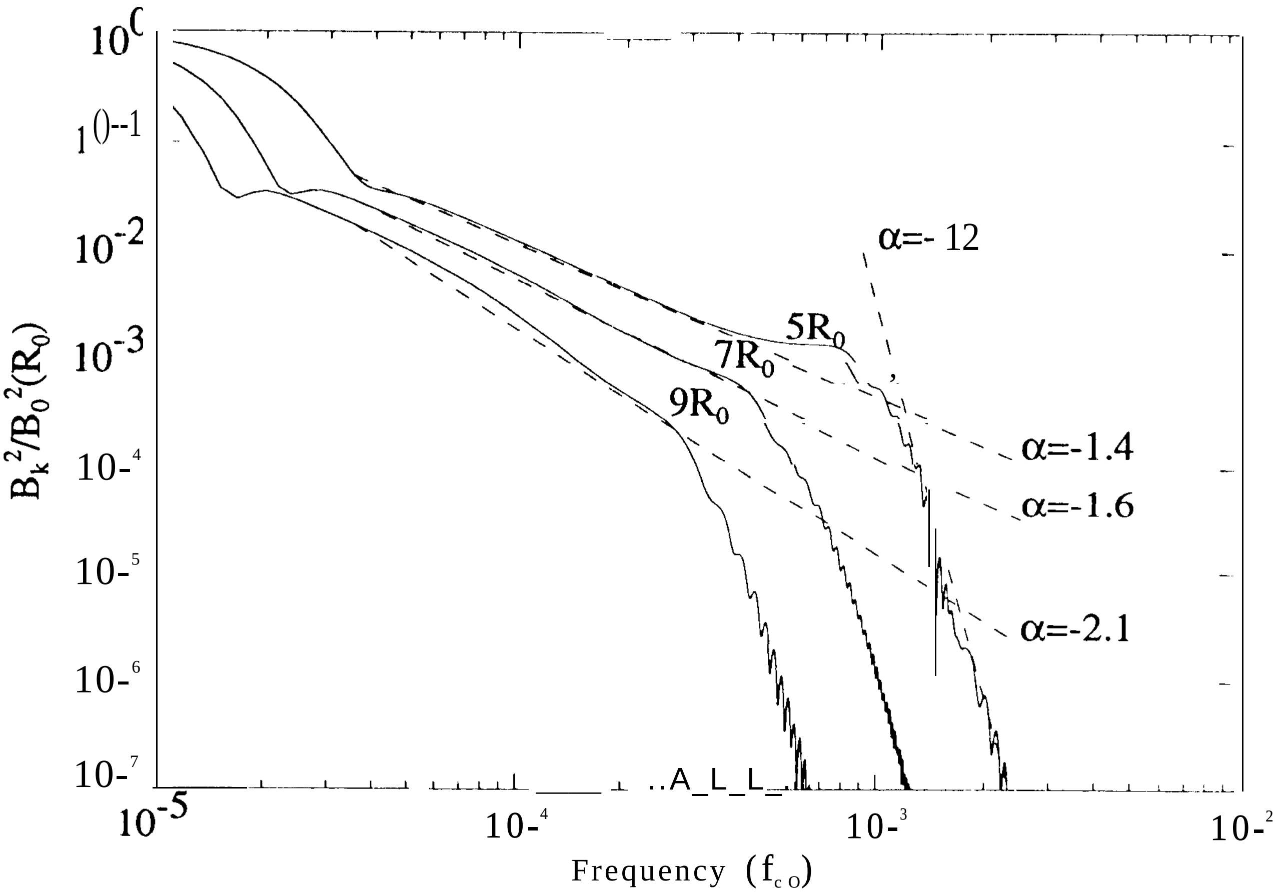
10°
10-2
10-4
10-6
10-8
$B_k^2/B_0^2(R_0)$
α=-12
5R0
7R0
9R0
α=-1.6
α=-2.1
α=-2.6
10-5
10-4
10-3
10:2
10-1
Frequency (f_{cO})

Fig 2.b



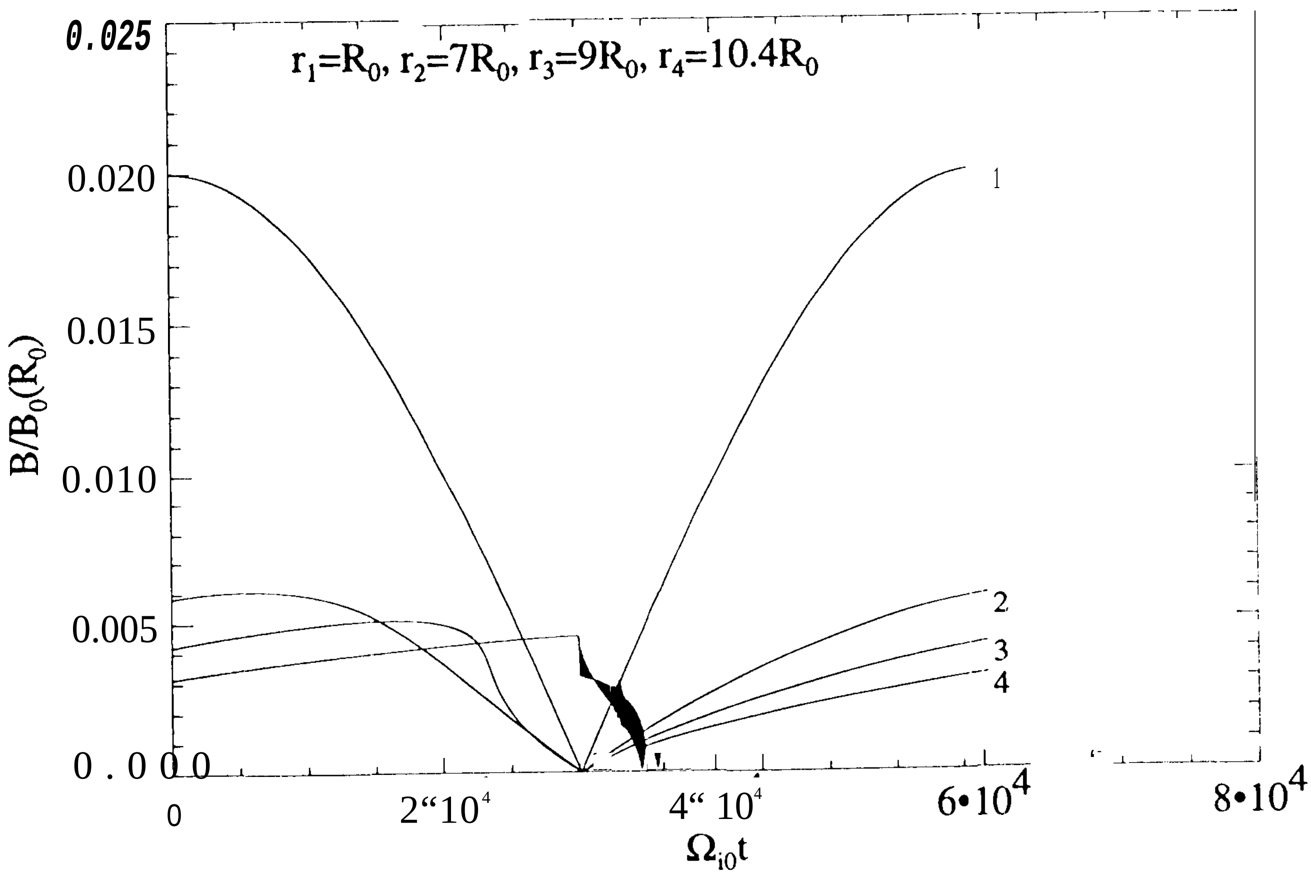
0.020
0.015
0.010
0.005
0.000
B/B0(R0"
r1=R0, r2=3.5R0, r3=5R0, r4=7R0, r5=9R0
1
2
3
4
5
0"
2"10^5
4"10^5
6o10^5
8•10^5
Ωi0t

Fig. 3a



10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
B_k^2/B_0^2(R_0)
α=- 12
5R0
7R0
9R0
α=-1.4
α=-1.6
α=-2.1
10^-5
10^-4
10^-3
10^-2
Frequency (f_co)

Fig. 3b



0.025
0.020
0.015
0.010
0.005
0.000
B/B0(R0)
r1=R0, r2=7R0, r3=9R0, r4=10.4R0
1
2
3
4
0
2·10^4
4·10^4
6·10^4
8·10^4
Ωi0t

Fig. 4